Can We Set Syllogistic Free?

PREFACE
HISTORICAL PART
SYSTEMATICAL PART
SYLLOGISTICS
1. Wide Syllogistic (W-S)
2. Aristotle’s Syllogistics (A-S)
3. Hispani Syllogistics (H-S)
4. Ockham’s Syllogistics (O-S)
5. Pauli Veneti Syllogistics (V-S)

K-INTERPRETATION
DECISION PROCEDURES
OBVERSION
CONVERSION
OBVERSION OF THE RESULT OF CONVERSION
PARTIAL CONTRAPOSITION
COMPLETE CONTRAPOSITION
PARTIAL INVERSION
COMPLETE INVERSION
SQUARE
SUBORDINATION
CONTRACTION
CONTRARIETY
SUBCONTRARIETY
SYLLOGISMS
FIGURE I
FIGURES II, III, IV

COMPARISON
MODELS FOR SYLLOGISTICS
PRELIMINARIES
$S_{SEN}$-LANGUAGES
$S_{EXP}$-LANGUAGES
$S_{FOR}$-LANGUAGES
RECAPITULATION

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I would be grateful for any suggestions and remarks.
Gyula Klima in his paper on reference and existence in mediaeval philosophy presents a quite intuitive interpretation of categorical propositions, which allows the rules of the Square of Opposition to hold without the assumption of the non-emptiness of common names. The main purpose of this paper is to develop this interpretation, definitely decide which syllogistic rules hold both in non-empty and empty domains of objects named, both with the assumption of non-emptiness of common names, and without it, and to compare the rules thus obtained with some approaches to the Syllogistic of categorical propositions. The secondary purpose is to show that there are some passages, as well in Aristotle’s Οργάνων, as in some works of other logicians, that the nowadays commonly accepted interpretation of Syllogistics fails to explain, and which are understandable on the ground of K-interpretation. The third purpose is to define some groups of syllogistic languages and to give definitions of model, K-model, satisfaction, truth and validity for this languages.

It is important that the assumption of non-emptiness of common names (Let us denote this assumption by „AN“) is not equivalent to the assumption of non emptiness of domain of objects named (Let us denote this assumption by „AD“).

Let the domain of objects named be U.

For every common name, there exists the set of its designata.

Then, when no existence-assumption is made, the family of sets containing the extensions of the elements of the range of common name variables is $2^U$.

Thus:

$AN = \{B \in 2^U \mid B \neq \Phi\}$ (only names with non-empty extensions are allowed), whereas

$AD = U \neq \Phi$ (only a non-empty domain is allowed).

AD and AN are obviously logically independent, as far as we do not restrict ourselves to treat Syllogistics as a systm given together with interpretation. However, it will become clear that, when we come down to semantical treatment which assumes that the variables range over names, all rules of inference in assertoric Syllogistics that hold without AN, also hold without AD.

HISTORICAL PART

I think it useful to distinguish at least two periods in the history of functional calculus approach to Syllogistics.

a) During the first one logicians applied functional calculus to Syllogistics without additional presumtions according to the interpretation:

$$SaP = \exists x[\neg x(S(x) \land P(x))] \equiv \forall x[S(x) \rightarrow P(x)]$$

$$SiP = \exists x[S(x) \land P(x)]$$

$$SeP = \exists x[S(x) \land \neg P(x)]$$

$$SoP = \exists x[S(x) \land P(x)]$$


3 Neither AD implies AN, nor AN implies AD. For it is possible that $U \neq \Phi$, and the family of allowed extensions of names is not restricted to $\{B \in 2^U \mid B \neq \Phi\}$, for $\forall A[\Phi \subseteq A]$, and if it is so, $\Phi \in 2^U$ independently on whether $U \neq \Phi$ or not. It is also possible that both the family of allowed extensions of names is $\{B \in 2^U \mid B \neq \Phi\}$ and $U = \Phi$, while $\{B \in 2^U \mid B \neq \Phi\} = \Phi$.

4 In every interpretation, if a term is infinite (I denote it by „.’“ added after it, e.g. „.’ P” “), then negation to the corresponding atom formula is added, e.g. in weak interpretation $SaP = \forall x[S(x) \rightarrow \neg P(x)]$.

5 This interpretation will be from now on called WEAK interpretation, as contrasted with the STRONG interpretation:

$$SaP = \exists x[S(x) \land \forall x(S(x) \rightarrow P(x))]$$

$$SiP = \exists x[S(x) \land \forall x(S(x) \rightarrow P(x))]$$

$$SeP = \exists x[S(x) \land \forall x(S(x) \rightarrow \neg P(x))]$$

$$SoP = \exists x[S(x) \land \neg P(x)]$$
After such a translation, numerous rules of inference in Syllogistics appeared to be invalid. Thus, Syllogistics was criticised.

b) Later on, the value of Syllogistics begun to be appreciated on the ground of its validity when some presumptions was made; e.g. Keynes proved that Syllogistics works if we assume that there are no empty classes, and if we forbid the usage of empty common names. Similarly, Ajdukiewicz proved that it works on the only presumption that there exist at least three different objects. And so on, the system has been rescued on the ground of WEAK or STRONG interpretation, with the help of existential presuppositions.

However, the case is not so simple.

Aristotle writes in his *Katgoriai* \(^6\) what follows:

\[\begin{align*}
\text{ἐπὶ δὲ γε τῆς καταφάσεως καὶ τῆς ὁποφάσεως ἄει, ἕν τε ἢ ἑαν τε μὴ ἢ, τὸ μὲν ἔτερον ἐσται ψεύδος τὸ δὲ ἔτερον ἀληθὲς· τὸ γὰρ νοσεῖν Σοκράτη καὶ τὸ μὴ νοσεῖν Σοκράτη, ὄντος τε αὐτοῦ φαινον ὅτι τὸ ἔτερον αὐτῶν ἀληθὲς ἢ ψεύδος, καὶ μὴ ὄντος ὅμοιος· τὸ μὲν γὰρ νοσεῖν μὴ ὄντος ψεύδος, τὸ δὲ μὴ νοσεῖν ἀληθὲς.}
\[Cat. 13b 27-33]\]

However, in the case of affirmation and negation, always, if [the object] is , and if [it] does not, one is false and the other true; For, of those [two:] that Socrates is ill and that Socrates is not ill, when Socrates exists, it is obvious that [exactly] one is true or [exclusive] false.\(^7\) And similarly, when he does not exist: for that he is ill when he does not exist is false, and that he is not ill, is true;\(^8\)

Let us analyse this statement with the use of functional calculus and WEAK interpretation.

We represent Socrates by constant: \(s\). Being ill by a predicate: \(C\). Thus, the sentence: ‘Socrates is ill’ is: \(C(s)\). Next, we define a predicate \(P\):

\[P(x) = x=s\]

It is clear that:

\[C(s) = \exists x [P(x) \land C(x)]\]

Accordingly:

\[\neg C(x) = \neg \exists x [P(x) \land C(x)]\]

And since

\[\neg \exists x [P(x) \land C(x)] = \forall x [P(x) \rightarrow \neg C(x)] = ^9 \text{PeC}\]

and

\[\text{PeC} \rightarrow \text{PoC} \quad (\text{Square of Oppositions})\]

\[\text{PoC} = \exists x [P(x) \land \neg C(x)] \quad (\text{still, WEAK interpretation})\]

we obtain:

\[\neg C(s) \rightarrow \exists x [P(x) \land \neg C(x)]\]

\(^6\) I decided to put titles in the cases indicated by their contexts.,

\(^7\) Or we may say more clearly, less faithfully: when Socrates exists, exactly one is true and the other false.

\(^8\) The list of editions of original texts used is in the bibliography.

\(^9\) According to WEAK interpretation.
Now, consider the situation described by Aristotle:

a) Socrates does not exist. (καὶ μὴ ὄντος ὁμοίος)
b) ‘Socrates is ill’ is false. (τὸ μὲν γὰρ νοσεῖν μὴ ὄντος ψεῦδος)
c) ‘Socrates is not ill’ is true. (τὸ δὲ μὴ νοσεῖν ἀληθεύει)

The proposition b) is quite coherent with the WEAK interpretation. But a) and c) are not. For, if c) is true, then ∃x[P(x)∧¬C(x)]. If so, it is also true that ∃xP(x), and therefore ∃x(x=s), Socrates exists. But, according to a), he does not. Thus, we have contradiction.

It should be clear, that application of STRONG interpretation also results in contradiction.10

The argumentation given above might meet with the response that the authority of the late part of Καταγροσιῶν is dubious, and Aristotle’s understanding of essence required the actual existence of the object essence of which is taken under consideration. To support this objection, Aristotle himself may be quoted:

’Ετι πῶς δείξει τὸ τί ἐστιν; ἀνάγκη γὰρ τὸν εἰδότα τὸ τί ἐστιν ἀνθρώπος ἢ ἄλλο ὁποῖον, εἰδέναι καὶ ὅτι ἐστιν (τὸ γὰρ μὴ ὁνομαίσθαι ὅδεν ὃ τί ἐστιν,
[An. Post. 92b 4-7]

Moreover, how [one] proves the essence? For it is necessity that one who knows the essence11 of human, or fo anything else, knows also that [it] is. (For [there is] no one who knows the essence of what is not...

Yes. I agree that Aristotle claimed that it is impossible to know the essence without knowing about existence. But, if it is to be a ground for an objection that Syllogistics assumed non-emptiness of terms: non sequitur. For aristotelian essence, strictly speaking is ‘what something is’ (τὸ τί ἐστιν). If we remember about it, it seems more understandable that he claimed that we cannot know what something is if it (generally) is not. However, it has very little importance for the problem we investigate. For Aristotle considered the usage of terms without knowing the essence of its hypothetical designates quite legitimate. He distinguished simply between definition of a term and the cognition of an essence of a being. Thus, the passus just quoted goes on as follows:

(τὸ γὰρ μὴ ὁνομαίσθαι ὅδεν ὃ τί ἐστιν, ἀλλὰ τί μὲν σημαίνει ὃ λόγος ἢ τὸ ὅνομα, ὅταν εἴπω τραγέλαφος, τί δ’ ἐστι τραγέλαφος ἀδύνατον εἰδέναι).
[An. Post. 92b 7-8]

(For [there is] no one who knows the essence of what is not, but [one may know] what an expression or name means, for I can say ['] buck-stag ['], but it is impossible to know the essence of buck-stag.

This leads us to a more general consideration. Syllogistics was believed to work when names replaced the variables. Therefore, if we discover that Aristotle or any of his followers argues against the usage of empty names, we may say that consequenter, he argues against empty names in Syllogistics. Et e converso if we see that he has nothing against the usage of empty names, and moreover, argues that they may be used and that they have meaning, we have strong suggestions, that, as far, as he does not explicate any restrictions, he has nothing against the usage of empty names in Syllogistics. This is the reason for which we shall stop for a while to see that Aristotle (and not only he, but we will back to this issue soon) claimed that in scientific theory terms may be defined and used independently on whether their designata exist.

Aristotle believed definitions to consist a quite important part of a theory (see e.g. An. Post. 90b). What did he say about the relation between definition and existence?

10 For C(s) = ∃x[P(x)∧C(x)] = PiC. Now, it is not true that Socrates is ill. Therefore, it is not the case that PiC. Two subcontraries cannot both be false. Hence, it is the case that PoC. Since (in STRONG interpretation) PoC≡∃x[P(x)∧¬C(x)], ∃x P(x) and Socrates exists. Which, according to Aristotle, is not the case.
11 Nota bene, I translate here ‘τὸ τί ἐστιν’ as ‘essence’. However it may be a common practice, it must be remembered that aristotelian essence may have some features very different from those ascribed to essences by e.g. Husserl, or some other philosophers.
Knowing by some definition what it is, it is unknown whether it is.

And also:

Φανερὸν δὲ καὶ κατὰ τοὺς νῦν τρόπους τῶν ὁρῶν ὡς οὐ δεικνύουσιν οἱ ὁριζόμενοι ὃτι ἑστὶν.

[An. Post. 92b 19-20]

And it is clear that, according to nowadays used modes of definitions, those who define do not show that [designatum definiendi] exists.

Thus, according to Aristotle, we may introduce to theory terms without knowing whether their designata exist. The role that Syllogistics played in his methodology strongly suggests that terms defined could take their place in syllogistic reasonings.12

However, considerations of this issue are not limited only to Aristotle himself. We can meet through whole the middle ages not only syllogistic reasonings obviously using empty terms, but also theoretical investigations about the relation between content of a name and the existentia designatorum. Let us look at the former before coming to the latter.

Let us take under consideration Sophismata written by Wilhelm of Heytesbury. In 26-th sophismato he mentions the following syllogism:

omne quod fuit est; fenix fuit; ergo fenix est.

[Heytesbury, Soph. 146rb]

Everything that was, is. Fenix was. Therefore fenix is.

For us, the point is not whether there is a mistake. It is the fact that in toto sophismato Heytesbury never says: This syllogism is erroneous, for it uses an empty term. No. Moreover, he almost explicitly states that we can use terms independently on whether ist designata exist. For he writes:

...assumpt ... quod quaelibet talis negativa particularis indefinita vel singularis in qua praedicatur hoc verbum ‘est’ secundum adjacens est impossibilis et includens contradicionem in se, cujusmodi sunt istae ‘Caesar non est’, ‘chimaera non est’, ‘hoc non est, demonstrato Socrate vel Platone vel quocumque alio’, ‘alia fenix non est’, et sic de omnibus talibus. Sed haec responsio dignissima est derisu et insipida...

[Heytesbury, Soph. 146va]

......we assume ... that any such negative particular indefinite or singular [proposition], in which the word ‘is’ is predicated as prædicum [not as a connective], is impossible and including contradiction... [Propositions] of this kind are: ‘Caesar is not’, ‘Chimera is not’, ‘It is not’, when we demonstrate Socrates or Plato or anything else, ‘Some fenix is not’, and so about any such [propositions]. But this answer is one the most ridiculous and stupid...

To give an another example. Paulus Venetus in his handbook Logica Parva writes:

...haec est vera ‘Aliqua rosa non est substantia’ nulla rosa existente. Et tamen haec est falsa ‘Aliqua non substantia non est non rosa’, quia suum contradictorium est verum, videlicet ‘Omnis non substantia est non rosa’...

[Venetus, LP, I, 36, p.10 v.28-31]

...this [proposition] is true: ‘Some rose is not a substance’, when no rose exists. But this [proposition] is false ‘Some non-substance is not a non-rose’, for its contradictory [proposition] is true, namely ‘Every non-substance is non-rose’...

12 It seems that Aristotle did use similar expressions to name two things: the essence as that, what actually consists the formal cause of an object, and the meaning of its name (i.e. what something is according to definition).
Let us concentrate for a while on this reasoning in order to estimate in on the ground of WEAK or STRONG interpretation. What Paul says, looks in functional calculus as follows (WEAK interpretation):

\[ \forall x (\neg \exists x (S(x) \rightarrow \neg R(x))] \]

Therefore, ist contradictory: ‘Some non-substance is not a non-rose’ is false.

\[ \neg \exists x (\neg S(x) \land \neg R(x)) \]

Hitherto, everything seems to be acceptable. There are substances, there are non-substances. The first proposition is true, since every existing rose is a substance. But Paul adds:

If there is no rose, ‘Some rose is not a substance’ is true.

If \[ \neg \exists x R(x) \] then \[ \exists x [R(x) \land \neg S(x)] \]

By distributing \[ \exists \] and omission of conjunction in the consequens we obtain an expression which seems to be false:

If \[ \neg \exists x R(x) \] then \[ \exists x R(x) \]

The same difficulty arises, when we apply STRONG interpretation, for the interpretation of ‘Some rose is not a substance’ is exactly the same.

Our Venetian Logician writes also:

.. non sequitur ‘Chimaera quae currit non movetur; ergo chimaera currit’ quia antecedens est verum, et consequens falsum....

[Venetus, LP, III, 8, p. 55, v. 2-4]

...there is no consequence: ‘Chimera which runs, does not move. Therefore chimera currit’, because the antecedens is true, and the consequens is false...

Let us treat ‘Chimera which runs’ as an name-expression. ‘Chimera which runs, does not move.’is a negative sentence. ‘Chimera runs’ - a positive one. How Paul of Venice could have known which is false and which is not? Supposedly, though a scientist he was, he haven’t been observing chimeras, and noting, whether running chimeras move or not. I guess that he holded the belief that there are no chimeras, without investigating whether they run or not. But if he did so (i.e. holded the belief), we have a situation similar to that of ill non-existing Socrates in Κατηγορίες. Namely, negative sentence (which implies some negative particular sentence with empty subject) is true. It cannot take place, when either WEAK or STRONG interpretation are sound. Positive sentence, though, with empty subject, is false, just as if positive sentences had existential import, and the negative ones had not.

Coming back to the more theoretical grounds, which we reffered to by speaking about the VII-th chapter of An. Post.; There are two commentaries on this chapter written by two quite prominent representatives of mediaeval philosophy, who are sometimes even opposed to each other. Namely Thomas Aquinas and Duns Scotus. Each of them has written a commentary on Αναλυτικόν Τοπές. Both seem to agree with Aristotle. Thomas writes:

Non est ergo possibile quod eadem demonstratione demonstret aliquis quid est et quia est. [Aquinas, Expositio Posteriorum, lib. 2 l. 6 n. 3 ]

Therefore, it is not possible for anyone to prove by the same proof what [something] is, and that [it] is.

Indeed, he uses this claim in practice, when he argues against the now so called ‘ontological proof’

I find it quite awkward that the proof ‘going’ from words to things is called ‘ontological’, and not ‘logoontical’.

\[\text{[13]}\]
Even if anyone understood that by this name: ‘God’ is signified what has been told, namely something such that nothing greater than it can be thought of, it would not imply that [anyone] understands that this, what is by this name signified, really exists...

Quite interesting, Johannes Duns Scotus devotes a particular question to the difficulty: Utrum quaestio quid est praesupponat si est? His answer is somewhat more complicated:

...quod quid est non praesupponit esse essentiae...nec praesupponit esse existere: quia esse existere est primo ipsius singularis...sed quid est praesupponit tertium esse, quod est actualiter entis... et illud esse quod est actualiter entis nihil aliud est nisi quidam gradus essendi unius essentiae distinctus contra alium gradum alterius essentiae...et illud esse est quodammodo non esse prohibitum in rerum natura; sive esse in habitu... et non esse existere...

[In Ar. Log., q. LII., p. 460]

What [something] is pressuposes neither existence of essence...nor existential existence [...]: for existential existence is firstly proper to a singular itself...but what [something] is pressuposes third existence, which is of actually being...and this existence which is nothing else that some grade of existence of one essence distinguished from the other grade of another essence...and this existence is in some way non-existence prohibited in reality; or existence in habitu...and not the existential existence.

Scottus uses quite a complicated language. I, after reading his commentary, am non sure what esse actualiter entis may be. Nevertheless, it seems that it is not the actual existence of particular designata. Firstly, because the existence of singulars is called esse existere: Secondly, because it does not seem probable that the existence of singular designatorum is something which may be described by saying: this existence which is nothing else that some grade of existence of one essence distinguished from the other grade of another essence...Thirdly, because Scottus writes that the question: what [something] is...

...praesupponit enim aliquod tertium esse quod nec est esse essentiae nec esse existere quod mensuratur tempore... [In Ar. Log., q. LII., p. 461]

...presupposes some third existence which neither is the existence of essence, nor the existential existence which is measured by time...

I am also quite unsure, how to understand the expression ‘existence measured by time’. The first interpretation that comes to my mind is: ‘existence in time’. If it is correct, then the most probable interpretation of his (I mean Scotus) view is that esse existere which is the existence of particular being is not pressuposed by what [something] is; the only existence presupposed is some esse actualiter entis of which we know that it is: atemporal, some kind of possibility of real existence (esse existentiae in habitu?), and some grade of existence of one essence distinguished from the other grade of another essence.

To recapitulate the historical part of this paper: There are some passages, in Aristotle and in mediaeval logicians, which are incoherent with the two nowadays accepted interpretations of assertoric Syllogistics. On the grounds of their (logicians) own words it would be somewhat illegitimate to consider that they excluded from Syllogistics the usage of empty terms.
SYSTEMATICAL PART

SYLLOGISTICS

By „Syllogistics“ in this paper I denote only that part of mediaeval logic which is concerned with simple, categorical propositions and relations between them.(the term is known enough to make the definition redundant; the emphasis is only put to exclude analysis of modalities or intensional contexts)

There are, respectively to the rules of inference accepted, at least a few syllogistics. Let us distinguish, then.\(^\text{14}\)

All syllogistics use the rule:

\[
\varphi_1 \quad \psi \quad \varphi_1
\]

If \(\varphi_n\) and \(\chi\) then \(\varphi_n\)

\[
\psi \quad \chi
\]

to allow mediate inference. Thus, I will concentrate on the basis of immediate inferences accepted in different versions of syllogistics.

1. Wide Syllogistic (W-S)

To denote the fullest (containing the longest list of immediate inferences) syllogistic, I use the abbreviation: „W-S“. W-S consists of following rules of immediate inference:

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<th>(\text{SiP} )</th>
<th>(\text{SeP} )</th>
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- all rules of the Square of Oppositions
- all syllogisms:
  - Barbara, Celarent, Darii, Ferio
  - Cesare, Camestres, Festino, Baroco
  - Darapti, Disamis, Datisi, Felapton, Bocardo, Ferison
  - Bamalip, Camenes, Dimatis, Fesapo, Fresison\(^\text{15}\)

\(^{14}\) In the presentations of syllogistics below some simplifications have been committed; However, in any syllogistic besides W-S I tried to list those rules which both: were accepted by a logician and belong to W-S.

\(^{15}\) In this paper I will not differentiate syllogistics with regard to the differences in presentation of syllogisms (three or four figures, etc), or in the presentation of the square of oppositions., since for my purpose such differentiation would be irrelevant. (The final list of syllogisms and rules of the square is the same in all cases.)
2. Aristotle’s Syllogistics (A-S)\textsuperscript{16}

- Conversion of $\text{SaP}$ (partial), of $\text{SiP}$ (complete) and of $\text{SeP}$ (complete)
- all rules of the Square of Oppositions\textsuperscript{17}
- all syllogisms

3. Hispani Syllogistics (H-S)\textsuperscript{18}

It consists of following rules of immediate inference:

Simple conversion of $\text{SeP}$ and $\text{SiP}$
Conversion per accidents of $\text{SeP}$ and $\text{SaP}$
Complete contraposition of $\text{SaP}$ and $\text{SoP}$

Square of Oppositions
syllogisms

4. Ockham’s Syllogistics (O-S)\textsuperscript{19}

Simple conversion of $\text{SeP}$ and $\text{SiP}$.
Conversion per accidents of $\text{SaP}$

Square of oppositions
syllogisms

5. Pauli Veneti Syllogistics (P-S)\textsuperscript{20}

Simple conversion of $\text{SeP}$, $\text{SiP}$
Conversion per accidents of $\text{SaP}$
Contraposition of $\text{SaP}$, $\text{SoP}$
Obversion (at least of $\text{SaP}$ and $\text{SeP}$)\textsuperscript{21}

Square of Oppositions
syllogisms

\textsuperscript{16} It is especially explained in \textit{An. Pr.}. Quite a precise presentation is also to be find in: Bocheński, I. M., „Ancient Formal Logic“, North-Holland Publishing Company, Amsterdam 1957, pp.42-54, and in: Łukasiewicz, Jan, „Syllogistyka Arystotelesa z punktu widzenia współczesnej logiki formalnej“, PWN, Warszawa, 1988, pp.34-104

\textsuperscript{17} Square of Oppositions is described rather in his work Περὶ ἕρμηταις.

\textsuperscript{18} His Syllogistics is to be found in \textit{Summulis Logicalibus} Petri Hispani. (Tratatus „De propositionibus“, esp. I.18-I.21). I use polish translation by Tadeusz Włodarczyk, „Traktaty Logiczne“, PWN, W-wa 1969

\textsuperscript{19} The Syllogistics of Ockham is to be found in his \textit{Summa Logicae} (especially in the second book). Ockham’s treatment of conversion seems to me somewhat odd. First, he defines kinds of conversion - simplex, per accidents, et contraposition. Then, he gives rules for the first two only, leaving contraposition aside. Thus, I list only those rules, that were by Ockham explicitly stated. I use T. Włodarczyk’s translation „Suma Logiczna“, PWN, W-wa 1971.

\textsuperscript{20} The Syllogistics of Paul of Venice do not differ very much from those of Buridan, besides only one rule, given by Paul, and obtainable from the rules given by Buridan), to be found in Buridan’s \textit{Summulae de Dialectica} and in \textit{Logica Parva Pauli Veneti}. As to Buridan, I use the version I cannot quote or refer to. As to Paulus Venetus, I use the critical edition by Alan Perreiah, Brill, Leiden, 2002.

\textsuperscript{21} Strictly speaking, both, Buridan and Paul, give such rules of aequipollence: $\text{SaP}' = \text{SeP}$, $\text{SeP}' = \text{SaP}$, while in Buridan $\text{SaP}'$ and $\text{SeP}'$ seem to be examples of general rule. (In Buridan it is the second rule of aequipollence, Paul writes about it in I. tractatus, cap. 10 De aequipollentis).
K-INTERPRETATION

Gyula Klima in his paper: “Existence and Reference in Medieval Logic” presents quite different interpretation of truth-conditions of categorical propositions:

According to what historians of medieval logic dubbed the inherence theory of predication, an affirmative categorical proposition (in the present tense with no ampliation[...]) is true only if an individualized property (form, or nature) signified by the predicate term actually inheres in the thing(s) referred to by the subject term.

On the other hand, according to the other basic type of medieval predication theories, the so-called identity theory, an affirmative categorical proposition is true only if its subject and predicate terms refer to the same thing or things. For example, on this analysis 'Socrates is wise' is true if and only if Socrates, the referent of 'Socrates', is one of the wise persons, the referents of the term 'wise'. If any of the two terms of an affirmative categorical is "empty", then the term in question refers to nothing. But then, since "nothing is identical with or diverse from a non-being", as Buridan (the "arch-identity-theorist" of the 14th century) put it...

Further on he presents an account, which, I think, allows to construct the following interpretation of categorical propositions in functional calculus:

Further on he presents an account, which, I think, allows to construct following interpretation of categorical propositions in functional calculus:

SaP \equiv \exists x(S(x) \land \forall x(S(x) \rightarrow P(x))
SiP \equiv \exists x(S(x) \land P(x))
SeP \equiv \forall x(S(x) \rightarrow \neg P(x))
SoP \equiv \neg \exists x(S(x) \lor \exists x(S(x) \land \neg P(x))

Let us denote this interpretation K-Interpretation („K“ from „Klima“)

In his paper Klima has shown that in this interpretation the Square of Oppositions holds for empty common names as subjects. The purpose of the following deliberations is to provide at full length the decision: what rules of syllogistic hold for empty common names (as subjects and as praedicata), and also in empty domain of objects named, and to compare the resulting set of rules (let us call it F-S form „Free“) with W-S, A-S, H-S, O-S, and V-S introduced above.

DECISION PROCEDURES

In decision procedures I proceed according to the scheme below:

a) Instead of checking a rule, formulate the implication corresponding to it, which is to be checked (step based on deduction theorem).

b) Translate the implication from syllogistic to the functional calculus according to given K-Interpretation.

c) Check the validity of the obtained formula using 0-1 decision procedure for narrower 1-place predicate calculus.

---

22 Here, the question whether we interpret Syllogistics as a system of ruler, or as a system of theses, is irrelevant.
NOTES:
1. For any two predicates S, P, consider the following diagramme:

   ![Diagram](image)

   I denote the subsets of V: I, II, III, IV, correspondingly by: <1, 0>, <1, 1>, <0, 1>, <0, 0>.

2. For any three predicates S, M, P, consider the following diagram:

   ![Diagram](image)

   I denote the subsets of V: I, II, III, IV, V, VI, VII, VIII, correspondingly by: <1, 0, 0>, <1, 0, 1>, <0, 0, 1>, <1, 1, 0>, <1, 1, 1>, <0, 1, 1>, <0, 1, 0>, <0, 0, 0>.

3. To denote the emptiness (non-emptiness) of one of above mentioned sets, I use the common notation, e.g.:
   <1, 0, 0> = Φ ( <1, 0, 1> ≠ Φ).

4. I assume that the reader is acquainted with 0-1 decision procedure for narrower 1-place predicate calculus.

5. In the procedure I use underlining to denote non-emptiness, outlining to denote emptiness, and bold to denote assumption. Contradiction by ‘c.’.

6. The commentary that ‘formula ψ is falsified when φ’ is to be understood: φ is sufficient (not necessarily necessary) condition for formula ψ to be falsified.

**OBVERSION**

a) \(SaP \rightarrow SeP’\)

\[\exists x \ S(x) \land \forall x \ (S(x) \rightarrow P(x)) \rightarrow \forall x \ (S(x) \rightarrow \neg \neg P(x))\]

\[
\begin{array}{cccccccc}
1 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

This formula is valid.

b) \(SiP \rightarrow SoP’\)
\[
\exists x \ (S(x) \land P(x) \rightarrow \neg \exists x \ (S(x) \lor \neg P(x))
\]

This formula is valid.

c) \(SeP \rightarrow Sa'\)

\[
\forall x \ (S(x) \rightarrow \neg P(x) \rightarrow \exists x \ (S(x) \land \forall x \ (S(x) \rightarrow \neg P(x)))
\]

This formula is invalid. It is falsified when \(<1, 1>=Φ<>1, 0>\).

d) \(SoP \rightarrow Si'\)

\[
\neg \exists x \ (S(x) \lor \exists x \ (S(x) \land \neg P(x))) \rightarrow \exists x \ (S(x) \land \neg P(x))
\]

This formula is invalid. It is falsified when \(<1,0>=Φ<>1,1>\).

**CONVERSION**

a) \(SaP \rightarrow PiS\)

\[
\exists x \ S(x) \land \forall x \ (S(x) \rightarrow P(x)) \rightarrow \exists x \ (P(x) \land S(x))
\]

This formula is valid.

b) \(SiP \rightarrow PiS\)
\[\exists x \ (S(x) \land P(x) \rightarrow \exists x \ (P(x) \land S(x))\]

This formula is valid.

c) \( SeP \rightarrow PeS \)

\[\forall x \ (S(x) \rightarrow \neg P(x)) \rightarrow \neg \exists x \ P(x) \lor \forall x \ (P(x) \rightarrow \neg S(x))\]

This formula is valid.

**OBVERSION OF THE RESULT OF CONVERSION**

a) \( SaP \rightarrow PoS' \)

\[\exists x \ (S(x) \land \forall x \ (S(x) \rightarrow P(x)) \rightarrow \neg \exists x \ P(x) \lor \forall x \ (P(x) \land \neg S(x))\]

This formula is valid.

b) \( SiP \rightarrow PoS' \)

\[\exists x \ (S(x) \land P(x) \rightarrow \neg \exists x \ P(x) \lor \forall x \ (P(x) \land \neg S(x))\]

This formula is valid.

c) \( SeP \rightarrow PaS' \)
∀x ((S(x) → ¬P(x)) → ∃x P(x) ∧ ∀x (P(x) → ¬S(x)))

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This formula is not valid. It is falsified when <1, 0>≈Φ≈<1, 0>.

PARTIAL CONTRAPOSITION

a) SaP → P′eS

∃x S(x) ∧ ∀x (S(x) → P(x)) → ∀x (¬P(x) → ¬S(x))

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This formula is valid.

b) SeP → P′iS

∀x (S(x) → ¬P(x)) → ∃x (¬P(x) ∧ S(x))

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This formula is not valid. It is falsified when <1, 1>≈Φ≈<1, 0>.

c) SoP → P′iS

¬∃x S(x) ∨ ∃x (S(x) ∧ ¬P(x)) → ∃x (¬P(x) ∧ S(x))

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This formula is not valid. It is falsified when <1, 1>≈Φ≈<1, 0>.

COMPLETE CONTRAPOSITION
a) \( SaP \rightarrow P'aS' \)

\[
\exists x \ S(x) \land \forall x \ (S(x) \rightarrow P(x)) \rightarrow \exists x \lnot P(x) \land \forall x \ (\lnot P(x) \rightarrow \lnot S(x))
\]

This formula is not valid. It is falsified when \(<1, 1> \neq \Phi, <1, 0> = \Phi = <0, 0>\).

b) \( SeP \rightarrow P'oS' \)

\[
\forall x \ (S(x) \rightarrow P(x)) \rightarrow \lnot \exists x \lnot P(x) \lor \exists x \ (\lnot P(x) \land \lnot S(x))
\]

This formula is not valid. It is falsified, when \(<1, 1> = <1, 0> = \Phi \land <0, 0> \neq \Phi\).

c) \( SoP \rightarrow P'oS' \)

\[
\lnot \exists x \ S(x) \lor \exists x \ (S(x) \land \lnot P(x)) \rightarrow \lnot \exists x \lnot P(x) \lor \exists x \ (\lnot P(x) \land \lnot S(x))
\]

This formula is not valid. It is falsified, when \(<1, 1> = <1, 0> = \Phi \land <0, 0> \neq \Phi\).

**PARTIAL INVERSION**

a) \( SaP \rightarrow S'oP \)

\[
\exists x \ S(x) \land \forall x \ (S(x) \rightarrow P(x)) \rightarrow \lnot \exists x \lnot S(x) \lor \exists x \ (\lnot S(x) \land \lnot P(x))
\]

This formula is not valid. It is falsified, when \(<1, 0> = <0, 0> = \Phi \land <1, 1> \neq \Phi\).
b) $SeP \rightarrow S'iP$

$$\forall x \ (S(x) \rightarrow \neg P(x)) \rightarrow \exists x \ (\neg S(x) \wedge P(x))$$

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This formula is falsified, when $<1, 1> = <0, 1> = \Phi$. Thus, it is not valid.

**COMPLETE INVERSION**

a) $SaP \rightarrow S'iP'$

$$\exists x \ S(x) \wedge \forall x \ (S(x) \rightarrow P(x)) \rightarrow \exists x \ (\neg S(x) \wedge \neg P(x))$$

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This formula is falsified, when $<1, 1> = \Phi$ and $<0, 0> = \Phi$. It is not valid.

b) $SeP \rightarrow S'oP'$

$$\forall x \ (S(x) \rightarrow \neg P(x)) \rightarrow \neg \exists x \ (\neg S(x) \wedge \neg P(x))$$

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This formula is falsified when $<1, 1> = \Phi$ and $<1, 0> = \Phi$. Hence, it is not valid.
SQUARE

SUBORDINATION

a) \( SaP \rightarrow SiP \)

\[
\exists x \ S(x) \land \forall x \ (S(x) \rightarrow P(x)) \rightarrow \exists x \ (S(x) \land P(x))
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This formula is valid.

b) \( \neg SiP \rightarrow \neg SaP \)

Formula is valid. Simple transposition of \( SaP \rightarrow SiP \).

c) \( SeP \rightarrow SoP \)

Formula is valid. \( SeP = \neg SiP \) (see contradiction, below). \( \neg SiP \rightarrow \neg SaP \) (see above), and since \( SaP = \neg SoP \) (see contradiction, below), \( \neg SaP \equiv SoP \).

d) \( \neg SoP \rightarrow \neg SeP \)

Formula is valid. Simple transposition of \( SeP \rightarrow SoP \).

CONTRADICTION

a) \( SaP=\neg SoP \)

\[
\exists x \ S(x) \land \forall x \ (S(x) \rightarrow P(x)) = \neg [\neg \exists x \ S(x) \lor \exists x \ (S(x) \land \neg P(x))]
\]

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This formula is valid.
b) \( \neg P \equiv \neg P \)

\[
\begin{array}{c}
\forall x \ (S(x) \rightarrow \neg P(x)) \equiv \neg \exists x \ (S(x) \land P(x)) \\
\end{array}
\]

This formula is valid.

**CONTRARIETY**

a) \( S \rightarrow \neg P \)

\[
\begin{array}{c}
\exists x \ S(x) \land \forall x \ (S(x) \rightarrow P(x)) \rightarrow \neg \forall x \ (S(x) \rightarrow \neg P(x)) \\
\end{array}
\]

This formula is valid.

b) \( \neg S \rightarrow \neg P \)
This formula is valid (\( S \rightarrow \neg \neg P \), simple transposition, omission of negations)

**SUBCONTRARIETY**

a) \( \neg P \rightarrow S \)

\[
\begin{array}{c}

\neg \exists x \ (S(x) \land P(x)) \rightarrow \neg \exists x \ S(x) \lor \exists x \ (S(x) \land \neg P(x)) \\
\end{array}
\]

This formula is valid.
b) $\neg SoP \rightarrow SiP$

Formula is valid. ($\neg SiP \rightarrow SoP$, simple transposition, omission of negations)

**SYLLOGISMS**

**FIGURE I**

a) Barbara  $MaP \land SaM \rightarrow SaP$

$$\exists x \ M(x) \land \forall x \ (M(x) \rightarrow P(x)) \land \exists x \ S(x) \land \forall x \ (S(x) \rightarrow M(x)) \rightarrow \exists x \ S(x) \land \forall x \ (S(x) \rightarrow P(x))$$

This formula is valid.

b) Celarent  $MeP \land SaM \rightarrow SeP$

$$\forall x \ (M(x) \rightarrow \neg P(x)) \land \exists x \ S(x) \land \forall x \ (S(x) \rightarrow M(x)) \rightarrow \forall x \ (S(x) \rightarrow \neg P(x))$$

This formula is valid.

c) Darii  $MaP \land SiM \rightarrow SiP$

$$\exists x \ M(x) \land \forall x \ (M(x) \rightarrow P(x)) \land \exists x \ (S(x) \land M(x)) \rightarrow \exists x \ (S(x) \land P(x))$$

This formula is valid.
d) Fermi: \( \text{MeP} \land \text{SiM} \rightarrow \text{SoP} \)

\[
\forall x \ (M(x) \rightarrow \neg P(x) \land \exists x \ (S(x) \land M(x)) \rightarrow \neg \exists x \ (S(x) \lor \exists x \ (S(x) \land \neg P(x))))
\]

This formula is valid.

**FIGURES II, III, IV**

All syllogisms from FIGURE I are valid over an empty domain. Now, the reduction of all other syllogisms is performed with use only of: rules of contradiction, conversion, and metathesis praemissarum. All those rules are valid over an empty domain. Therefore, the reduction is successful, and all syllogisms from FIGURES II, III, IV, are valid over an empty domain.

**COMPARISON**

Below, I construct a table in which a comparison of A-S, W-S, H-S, O-S, P-S, and F-S is conducted. By „+“ I denote the fact that a rule holds (however instead of rules I write corresponding laws - it makes it easier and brings no loss of accuracy), and that it does not hold by „-“.

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<th>H-S</th>
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20
MODELS FOR SYLLOGISTICS

PRELIMINARIES

Hitherto we have developed the idea given by Gyula Klima than there is an interpretation in which syllogistics may be considered as a free logic (i.e. a logic which works without a presumption of existence of any objects). I have shown which exactly commonly accepted syllogistic rules are valid in empty domain and compared the result with some classic representatives of syllogistics.\(^{23}\) Our present purpose is to define a model for syllogistics and syllogistics free in K-interpretation (hence K-MODEL), and the interpretation of a language of syllogistics in a model, on the grounds of formal semantics. Therefore, the present task is to provide mainly a notational (and notional) apparatus.

We introduce some metalanguage variables:

- \(\varphi, \psi, \varphi_1, \ldots, \varphi_n, \psi_1, \ldots, \psi_n\) represent sentences.
- \(\chi, \chi_1, \ldots, \chi_n\) represent propositional expressions (including sentences)
- \(\tau, \tau_1, \ldots, \tau_n\) represent propositional formulas
- \(\mu, \nu, \mu_1, \ldots, \mu_n, \nu_1, \ldots, \nu_n\) represent names
- \(\alpha, \beta, \alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_n\) represent name-expressions (including names)

**S\(_{\text{SEN}}\)**-LANGUAGES

We shall start by defining languages of assertoric sentences. They are languages without variables. We shall not distinguish between languages and its algebras of expressions.

\[
L_1 \in S_{\text{SEN}} = \{ L_1 = \langle W_{\text{SEN}}^{L_1}, N_{\text{SEN}}^{L_1}, a, e, o, i, \neg, \wedge \rangle \}
\]

where \(N_{\text{SEN}}^{L_1}\) is a set of names of \(L_1\), \(a, e, o, i\) are functors of category s/n, n, and \(\neg, \wedge\) are the classical functors of negation and conjunction. \(W_{\text{SEN}}^{L_1}\) is a set of sentences dependent of the set \(N_{\text{SEN}}^{L_1}\). It is the least set which fulfills the conditions (2) and (3)\(^{24}\):

\[
\begin{align*}
\mu, \nu \in N_{\text{SEN}}^{L_1} & \rightarrow [\mu \alpha \nu], [\mu e \nu], [\mu o \nu], [\mu i \nu] \in W_{\text{SEN}}^{L_1} \\
\varphi, \psi \in W_{\text{SEN}}^{L_1} & \rightarrow [\neg \varphi], [\varphi \wedge \psi] \in W_{\text{SEN}}^{L_1}
\end{align*}
\]

The least set which fulfills only the condition (2) is the set of categorical sentences: \(\text{CAT}_{\text{SEN}}^{L_1}\).

The set \(W_{\text{SEN}}^{L_1} - \text{CAT}_{\text{SEN}}^{L_1}\) is a set of hypothetical sentences: \(\text{HYP}_{\text{SEN}}^{L_1}\).

\[
\text{If the model fulfills additionally the following conditions (5-8):}
\]

For all \(A, B \in 2^\text{Ob}\):

\[
\begin{align*}
(5) & \quad A \cap B = A \neq \emptyset \land A \subset B \\
(6) & \quad A \cap B = A \cap B = \emptyset \\
(7) & \quad A \cap B = A \cap B = \emptyset \\
(8) & \quad A \circ B = A \circ B = A \circ B = \emptyset
\end{align*}
\]

\(^{23}\) The K-interpretation is: SaP \(= \exists x S(x) \land \forall x (S(x) \rightarrow P(x))\), SiP \(= \exists x (S(x) \land P(x))\), SeP \(= \forall x (S(x) \rightarrow \neg P(x))\), SoP \(= \neg \exists x (S(x) \lor \exists x (S(x) \land \neg P(x)))\). Nevertheless, I am not going to discuss it in this paper. Here, it is assumed in the definition of F-model.

\(^{24}\) Symbols: a, e, o, i, used in metalanguage are names of the functors: a, e, o i.
the model is called K-model (since then it is a model of a syllogistics free in K-interpretation).

Let us assume that the model is fixed. Now we define the set of functions of denotation \( \text{DEN}_{L^1} \) mapping \( \text{N}_{\text{SEN}} \) into \( 2^{\text{Ob}} \):

\[
D^{L^1} \in \text{DEN}_{L^1} = \forall \mu \in \text{N}_{\text{SEN}}^{L^1} \exists! A \in 2^{\text{Ob}}[D(\mu) = A]
\]

We define the valuation of sentences in \( L^1 \): \( V_{\text{SEN}}^{L^1} \), when a \( D^{L^1} \) is fixed (since we have defined language quite syntactically, id does not have to). It is a function mapping \( \text{W}_{\text{SEN}}^{L^1} \) onto \{1, 0\}.

If \( \phi \in \text{W}_{\text{SEN}}^{L^1} \), then either \( \phi \in \text{CAT}_{\text{SEN}}^{L^1} \), or \( \phi \in \text{HYP}_{\text{SEN}}^{L^1} \).

If \( \phi \in \text{CAT}_{\text{SEN}}^{L^1} \), then it is of one of the forms: \( \begin{bmatrix} \mu a \nu \end{bmatrix} \), \( \begin{bmatrix} \mu e \nu \end{bmatrix} \), \( \begin{bmatrix} \mu i \nu \end{bmatrix} \), \( \begin{bmatrix} \mu o \nu \end{bmatrix} \).

We define \( V_{\text{SEN}}^{L^1} \) for elements of \( \text{HYP}_{\text{SEN}}^{L^1} \).

If \( \phi, \psi \in \text{CAT}_{\text{SEN}}^{L^1} \) then:

\[
\begin{align*}
V_{\text{SEN}}^{L^1}(\begin{bmatrix} \neg \phi \end{bmatrix}) = 1 & \equiv V_{\text{SEN}}^{L^1}(\phi) = 0 \\
V_{\text{SEN}}^{L^1}(\begin{bmatrix} \phi \land \psi \end{bmatrix}) = 1 & \equiv V_{\text{SEN}}^{L^1}(\phi) = V_{\text{SEN}}^{L^1}(\psi) = 1
\end{align*}
\]

We define \( V_{\text{SEN}}^{L^1} \) for elements of \( \text{HYP}_{\text{SEN}}^{L^1} \).

If \( \phi, \psi \in \text{CAT}_{\text{SEN}}^{L^1} \) then:

\[
\begin{align*}
V_{\text{SEN}}^{L^1}(\begin{bmatrix} \mu a \nu \end{bmatrix}) = 1 & \equiv D^{L^1}(\mu)aD^{L^1}(\nu) \\
V_{\text{SEN}}^{L^1}(\begin{bmatrix} \mu e \nu \end{bmatrix}) = 1 & \equiv D^{L^1}(\mu)eD^{L^1}(\nu) \\
V_{\text{SEN}}^{L^1}(\begin{bmatrix} \mu i \nu \end{bmatrix}) = 1 & \equiv D^{L^1}(\mu)iD^{L^1}(\nu) \\
V_{\text{SEN}}^{L^1}(\begin{bmatrix} \mu o \nu \end{bmatrix}) = 1 & \equiv D^{L^1}(\mu)oD^{L^1}(\nu)
\end{align*}
\]

Thus, an interpretation for \( L_1 \) belonging to \( S_{\text{SEN}} \) is:

\[
\exists \mathfrak{I}, \text{EXT}^{L^1}
\]

We can define truth for sentences in any \( L_1 \) belonging to \( S_{\text{SEN}} \), in an interpretation \( \mathfrak{I} \).

\[
T_{\mathfrak{I}}^{L^1}(\phi) = \text{EXT}^{L^1}(\phi) = V_{\text{SEN}}^{L^1}(\phi)
\]

\[
L_2 \in S_{\text{EXP}} = L_2 = \langle \text{W}_{\text{EXP}}^{L_2}, \text{N}_{\text{EXP}}^{L_2}, a, e, o, i, \neg, \land \rangle
\]

\[
\text{W}_{\text{SEN}}^{L_2} \subset \text{W}_{\text{EXP}}^{L_2} \\
\text{N}_{\text{SEN}}^{L_2} \subset \text{N}_{\text{EXP}}^{L_2}
\]

I do interpret the name ‘denotation’ exactly according to the definiton, though it may differ from the common use of this term.
where $N_{\text{SEN}}^{L_2} \neq \Phi$ is the set of all names in that language. $N_{\text{SEN}}^{L_2}$ is the basis of construction of $W_{\text{SEN}}^{L_2} \neq \Phi$, which is the least set satisfying the requirements given in (2), (3) (when ‘L1’ is replaced by ‘L2’), and (17-18), and

\[ N_{\text{EXP}}^{L_2} - N_{\text{SEN}}^{L_2} = \{\pi_1, \ldots, \pi_n\} \neq \Phi \]

where $\{\pi_1, \ldots, \pi_n\}$ are all name variables of a given language $L_2$, $\text{VAR}_{\text{EXP}}^{L_2}$.

\[ W_{\text{EXP}}^{L_2}, W_{\text{SEN}}^{L_2} = \{\tau_1, \ldots, \tau_n\} \neq \Phi \]

where $\{\tau_1, \ldots, \tau_n\}$ is the collectio omnium formularum of $L_2$: $\text{FOR}_{\text{EXP}}^{L_2}$ (to be defined below), and $W_{\text{EXP}}^{L_2}$ is the least set satisfying the conditions (17), (20-22)

\[ \forall \alpha, \beta \in \text{VAR}_{\text{EXP}}^{L_2}[\overline{\alpha \beta}], [\overline{\alpha \beta}], [\overline{\alpha \beta}] \in W_{\text{EXP}}^{L_2} \]

\[ \forall \chi, \mu \in W_{\text{EXP}}^{L_2}[\overline{\chi \mu}, \chi \mu \in W_{\text{EXP}}^{L_2}] \]

We define: $\text{FOR}_{\text{EXP}}^{L_2} = W_{\text{EXP}}^{L_2} \setminus W_{\text{SEN}}^{L_2}$. The least subset of the set $W_{\text{EXP}}^{L_2}; \text{CAT}_{\text{EXP}}^{L_2}$ which satisfies the condition (23):

\[ \forall \alpha, \beta \in N_{\text{EXP}}^{L_2}[\overline{\alpha \beta}], [\overline{\alpha \beta}], [\overline{\alpha \beta}] \in \text{CAT}_{\text{EXP}}^{L_2} \]

is the set of categorical propositional expressions of $L_2$. The set $W_{\text{EXP}}^{L_2} \cap \text{CAT}_{\text{EXP}}^{L_2} = \text{HYP}_{\text{EXP}}^{L_2}$ is the set of hypothetical expressions of $L_2$.

Furthermore, we can obtain the omnium categoricarum forunamurum collectio, which is: $\text{FOR}_{\text{EXP}}^{L_2} \cap \text{CAT}_{\text{EXP}}^{L_2}$, the omnium hypotheticarum formularum collectio, which is $\text{FOR}_{\text{EXP}}^{L_2} \cap \text{HYP}_{\text{EXP}}^{L_2}$.

We can also develop a notion of an expressional extension of a language belonging to $S_{\text{SEN}}$. Namely we can say that a language $L_2 \in S_{\text{EXP}}$ is an expressional extension of a language $L_1 \in S_{\text{SEN}}$ if

\[ W_{\text{SEN}}^{L_1} = W_{\text{EXP}}^{L_2} \land N_{\text{SEN}}^{L_1} = N_{\text{EXP}}^{L_2} \]

The notion of a model remains generally the same, as in the case of $S_{\text{SEN}}$-languages (with all the restrictions given to (4)):

\[ \mathcal{M} \text{ is a model for } L_2 \in S_{\text{EXP}} = \mathcal{M} =< \text{Ob}, \{1, 0\}, \alpha, \cdot, \circ, i > \]

The definition of denotation functions $\text{DEN}_{\text{L}}^{L_2}$ remains almost the same as in the case of $\text{DEN}_{\text{L}}^{L_1}$; they map $N_{\text{SEN}}^{L_2}$ into $2^{\text{Ob}}$:

\[ D_{\text{L}}^{L_2} \in \text{DEN}_{\text{L}}^{L_2} = \forall \mu \in N_{\text{SEN}}^{L_2} \exists ! A \in 2^{\text{Ob}}[D(\mu) = A] \]

the notion of valuation for sentences is quite similar to the notion of valuation for $S_{\text{SEN}}$-languages. It is a function mapping $W_{\text{SEN}}^{L_2}$ onto $\{1, 0\}$.

If $\phi \in W_{\text{SEN}}^{L_2}$, then either $\phi \in \text{CAT}_{\text{SEN}}^{L_2}$, or $\phi \in \text{HYP}_{\text{SEN}}^{L_2}$.

If $\phi \in \text{CAT}_{\text{SEN}}^{L_2}$, then it is of one of the forms: $[\overline{\mu \alpha \nu}], [\overline{\mu \nu}], [\overline{\mu \nu}], [\overline{\mu \nu}]$.

\[ V_{\text{SEN}}^{L_2}(\overline{\mu \alpha \nu}) = 1 = D_{\text{L}}^{L_2}(\mu \alpha D_{\text{L}}^{L_2}(\nu)) \]

\[ V_{\text{SEN}}^{L_2}(\overline{\mu \nu}) = 1 = D_{\text{L}}^{L_2}(\mu \nu D_{\text{L}}^{L_2}(\nu)) \]

\[ V_{\text{SEN}}^{L_2}(\overline{\mu \nu}) = 1 = D_{\text{L}}^{L_2}(\mu \nu D_{\text{L}}^{L_2}(\nu)) \]

\[ V_{\text{SEN}}^{L_2}(\overline{\mu \nu}) = 1 = D_{\text{L}}^{L_2}(\mu \nu D_{\text{L}}^{L_2}(\nu)) \]

We define $V_{\text{SEN}}^{L_2}$ for elements of $\text{HYP}_{\text{SEN}}^{L_2}$.

If $\phi, \psi \in \text{CAT}_{\text{SEN}}^{L_2}$ then:
(31) \[ V_{SEN}^{L^2}([\neg \phi]) = 1 \equiv V_{SEN}^{L^2}(\phi) = 0 \]
(32) \[ V_{SEN}^{L^2}([\phi \land \psi]) = 1 \equiv V_{SEN}^{L^2}(\phi) = V_{SEN}^{L^2}(\psi) = 1 \]

Now we define the property of sentence being true in a partial interpretation \( \mathfrak{M} = < \mathcal{M}, D^{L^2}, V_{SEN}^{L^2}> \):

\[ T_{\mathfrak{M}}^{L^2}(\phi) = V_{SEN}^{L^2}(\phi) = 1 \]

Some doubts arise, however, when we want to take under consideration name variables. For we can either emphasise that they are NAME variables, or that they are name VARIABLES. The question is: should we valuate a name variable via names, or not? If we choose the first option, consequently, we can allow only those valuation which can be ‘obtained’ by means of substitution of name variables by names as well. Namely, we must agree that, when we understand a valuation of name variables as a sequence \( \{A_1, ..., A_k\} \) of elements of \( 2^{Ob} \), we have to exclude from possible valuations such \( A_n \) for which \( \neg \exists \mu \in N_{SEN}^{L^2}[A_n = D(\mu)] \). On the other hand, if we choose the second option, we put no restriction of \( A_i \) contained by valuations, but accordingly concede that there are such valuations of name variables for which there are no corresponding names. We could avoid this difficulty by the simple assumption that any \( L_2 \in S_{EXP} \) is such, that

\[ \forall A_i \in 2^{Ob} \exists \mu \in N_{SEN}^{L^2}[A_i = D(\mu)] \]

Unfortunately, languages which do not fulfill this condition seem to be quite legitimate objects of investigation.

For convenience, we have decided to define valuation for \( S_{EXP} \) - languages by means of substitution, and to leave the most general concept of valuation for \( S_{FOR} \) - languages which do not contain names. There is no loss of accuracy, since all formulae valid in \( S_{EXP} \) - languages according to the former notion of valuation of name - variables are also valid according to the latter.

We define the complete-name-substitution. Only name-variables can be substituted (therefore ‘...-name-substitution’). All the variables in a formula are to be substituted (therefore it is ‘complete’).

\[ A \text{ formula } \tau, \text{ in which all name variables are } \pi_1, ..., \pi_k \text{ yields as the result of complete name substitution (}\pi_1/\mu_1, ..., \pi_k/\mu_k\text{) a sentence } \phi, \text{ which contains at least names: } \mu_1, ..., \mu_k \text{ iff the sentence } \phi \text{ differs from the formula } \tau \text{ only in having the names } \mu_1, ..., \mu_k \text{ in all those places in which the name-variables } \pi_1, ..., \pi_k \text{ respectively occur in } \tau. \]

We now define the valuations of all name - variables in an \( L_2 \in S_{EXP} \): \( V_{NV}^{L^2} \).

(36) If \( <\pi_1, ..., \pi_n> \) is the sequence of all elements of \( VAR_{EXP}^{L^2} \), then any \( k \)-place sequence \( s_k^{L^2} \) of names belonging to \( N_{EXP}^{L^2} \) is an element of \( V_{NV}^{L^2} \).

We can now characterise \( s_k \)-substitution (in other words: complete name substitution in valuation \( s_k^{L^2} \)), which is a kind of complete name substitution. If the sequence of all name-variables \( <\pi_1, ..., \pi_n> \) and the valuation \( s_k^{L^2} \) is fixed, then:

\[ A \text{ formula } \tau, \text{ in which all name variables are } \pi_1, ..., \pi_k \text{ yields as the result of } s_k^{L^2} \text{-substitution (}\pi_1/\mu_1, ..., \pi_k/\mu_k\text{) a sentence } \phi, \text{ which contains at least names: } \mu_1, ..., \mu_k \text{ iff the sentence } \phi \text{ differs from the formula } \tau \text{ only in having the names } \mu_1, ..., \mu_k \text{ in all those places in which the name-variables } \pi_1, ..., \pi_k \text{ respectively occur in } \tau \text{ and, for every } i, \text{ if } \pi_i \text{ is } m \text{-th in the sequence } <\pi_1, ..., \pi_n>, \text{ then } \mu_i \text{ is } m \text{-th in the sequence } s_k^{L^2}. \]
If \( s_k \) is given, we denote the result of \( s_k \)-substitution of \( \tau \) by ‘\( s_k(\tau) \)’. It remains to define the validarum formularum collectio of \( L_2 \): (VALEXP\(_{L_2}\)):

\[
\tau \in \text{VAL}_{L_2} \equiv \tau \in \text{FOR}_{L_2} \land \forall s_k \text{[T}_3
\]

The objects of our investigation are languages of assertorical syllogistics containing some name expressions. If a language fulfill this condition condition, then it either contains only names as name expressions, or both: names and name variables, or only variables. The first two laguages have been considered above. It remains to define languages with name variables and without names.

\( \text{SFOR}-\text{LANGUAGES} \)

\[
L_3 \in \text{SFOR} = L_3 = <\text{WFOR}_{L_3}, \text{NFOR}_{L_3}, a, e, o, i, \neg, \land>
\]

where restrictions regarding \( a, e, o, i, \neg, \land \) are the same, as in the case of (1), \( \text{NFOR}_{L_3} \) is the set \( \{n_1, ..., n_n\} \) of all name variables of \( L_3 \), and \( \text{WFOR}_{L_3} \) is the least set satisfying the conditions (39), (40):

\[
\pi_1, \pi_2 \in \text{NFOR}_{L_3} \rightarrow \left[ \pi_1a\pi_2 \right], \left[ \pi_1e\pi_2 \right], \left[ \pi_1i\pi_2 \right], \left[ \pi_1o\pi_2 \right] \in \text{WFOR}_{L_3}
\]

The least set satisfying onle the condition (39) is the collectio categoricarum formularum of \( L_3 \): CATFOR\(_{L_3}\).

The set: \( \text{WFOR}_{L_3} - \text{CATFOR}_{L_3} \) constitutes the collectio hypotheticarum formularum of \( L_3 \): HYPFOR\(_{L_3}\).

The definition of a model is almost the same, as in the cases above, with all restrictions added to (4):

\[
\mathcal{M} \text{ is a model for } L_3 \in \text{SFOR} \equiv \mathcal{M} = <\text{Ob}, \{1, 0\}, a, e, o, i>
\]

We define the set of valuations of name variables \( \text{VNV}_{L_3} \), which map \( \text{NFOR}_{L_3} \) into \( 2^{\text{Ob}} \):

\[
\text{If } <n_1, ..., n_n> \text{ is the fixed sequence of all elements of } \text{NFOR}_{L_3} \text{ then any } n\text{-place sequence } <A_1, ..., A_n> \text{ of elements of } 2^{\text{Ob}} \text{ is an element of } \text{VNV}_{L_3}.
\]

We now define the expression: ‘a formula \( \tau \) of \( L_3 \) is satisfied by the sequence \( s_i \) in \( \mathcal{M} \)', \( \text{STSF}_{s_i}^{L_3}(\tau) \):

First, we define this notion for \( \tau \in \text{CAT}_{L_3} \). If \( <A_1, ..., A_n> = s_i \), then:

\[
\tau = \pi_1a\pi_2 \rightarrow \text{STSF}_{s_i}^{L_3}(\tau)_{\mathcal{M}} = A_1aA_2
\]

\[
\tau = \pi_1e\pi_2 \rightarrow \text{STSF}_{s_i}^{L_3}(\tau)_{\mathcal{M}} = A_1eA_2
\]

\[
\tau = \pi_1i\pi_2 \rightarrow \text{STSF}_{s_i}^{L_3}(\tau)_{\mathcal{M}} = A_1iA_2
\]

\[
\tau = \pi_1o\pi_2 \rightarrow \text{STSF}_{s_i}^{L_3}(\tau)_{\mathcal{M}} = A_1oA_2
\]

We define the same notion for \( \tau \in \text{HYP}_{L_3}^{L_2} \):

\[
\tau = \left[ \neg\tau_1 \right] \rightarrow \text{STSF}_{s_i}^{L_3}(\tau)_{\mathcal{M}} = \neg \text{STSF}_{s_i}^{L_3}(\tau_1)_{\mathcal{M}}
\]

\[
\tau = \left[ \tau_1 \land \tau_2 \right] \rightarrow \text{STSF}_{s_i}^{L_3}(\tau)_{\mathcal{M}} = \text{STSF}_{s_i}^{L_3}(\tau_1)_{\mathcal{M}} \land \text{STSF}_{s_i}^{L_3}(\tau_2)_{\mathcal{M}}
\]

Next, we can define the expression: ‘the formula \( \tau \) of \( L_3 \) holds in the model \( \mathcal{M} \)', \( \text{HLD}_{L_3}^{L_2}(\tau)_{\mathcal{M}} \):

\[
26 \text{ The situation seems od: name variables without names. It nevertheless is not. There is an analogous phenomenon in propositional calculus, which usually is a calculus with propositional variables, but without propositions.}
\]

25
When this notion is given, the definition of the set of valid formulas of $L_3$ ($\text{VAL}_{\text{FOR}}^{L_3}$) is trivial.

\begin{align}
\text{RECAPITULATION} \\
\end{align}

There are some traces which suggest us that historically it is not obvious that Syllogistics took any kind of existential assumptions. As should be also clear, among all syllogistics mentioned, A-S and O-S remain sound on an empty domain. The syllogistic which differs most from F-S is V-S, which accepts at least four rules which do not hold in empty domain. It is, however, worth of mentioning, that the rules which all syllogistics agreed upon, i.e. conversion, square of opposites, and syllogisms hold firmly in empty domain in K-Interpretation. It is also possible to approach syllogistics with formal semantics ‘in hands’, and define their key notions.